Solutions to Some Mysteries of SCR Hydrodynamic Behavior and Their Applications for Improved SCR Design

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Abstract
Steel Catenary Riser (SCR) is the most cost effective option for numerous deep water field developments. Despite its structural simplicity, the design and analysis of SCR’s can be complex especially due to the hydrodynamic forces. This paper selects three of the unexplained observations in SCR analyses, solves the mysteries analytically, and provides recommendations accordingly for improving riser design.

The three selected observations that are not well understood occur at three distinct locations along the SCR: the hang-off, the middle section, and the Touch Down Zone (TDZ). The riser maximum stress at the hang-off can be lower when the wave particle motion is considered in the dynamic simulation compared with using vessel motion time trace only. The TDZ fatigue damage is observed to be lower when background current is applied along riser in the motion fatigue analysis, even if the current direction is perpendicular to the riser motion direction. Finally, the motion fatigue response is observed to be better with a shorter strake length compared with that of the configuration wherein the strake coverage is extended to the seabed.

The first two phenomena are generally explained by the “damping” effect, however, how the damping works was never clearly demonstrated. The third observation is actually contradictory to the damping effect as less strake length results in smaller drag forces and thus less damping. All of the above phenomena are related to the hydrodynamic forces that are exerted onto the riser by the surrounding sea water. Those forces are calculated by Morison’s equation which includes drag force and inertia forces. In this paper, a detailed analytical study using Morison’s equation illustrates how the drag force or inertia forces will affect the riser motion, and which force is the dominating one, for each of the three observations aforementioned.

The understanding of these riser hydrodynamic behaviors leads to improved riser design. The applications of wave hydrodynamics and background currents present more realistic riser responses and reduce conservatism and thus cost. A riser with shorter strake length but more fatigue life will result in reduced cost too provided that adequate VIV fatigue life is preserved.

Introduction
Due to deeper water depth, higher design pressures, vessels with more dynamic motions, and severe weather conditions, the design of risers in deep water applications has seen increasing challenges. Flexible risers are generally not sensitive to high porc motions, but are sometimes limited by pressure rating, temperature requirements and size. Single Line Offset Riser (SLOR) [1] or Free Standing Hybrid Riser (FSHR) [2] provide a feasible solution for a variety of applications, but to date these risers have been on the order of 2-3 times the cost an equivalent SCR. The use of Steel Catenary Risers (SCR) is often the preferred solution as it is by far the least expensive concept, and with it being an extension of the flowline, makes the installation more seamless.

Despite its structural simplicity, the design and analysis of SCR can be complex. Hydrodynamic forces are among some of the most challenging analysis considerations in the design. This paper selects three frequently misunderstood hydrodynamic behaviors in SCR dynamic responses, solves the mysteries analytically, and provides recommendations.
accordingly for improving riser design. The three selected observations occur at three distinct locations along the SCR: the hang-off, the middle section, and the Touch Down Zone (TDZ).

The riser hangoff response can be affected by the wave that is significant to a depth of roughly half the wavelength. If during a riser dynamic simulation the vessel motions are applied without the associated wave circulation, the hydrodynamic forces on riser will be incorrectly simulated. One observation that many riser engineers have is that the riser extreme response at the hangoff is much lower due to the existence of wave particle motions, given that the vessel motions are the same, especially in high sea states. For example, an SCR assessment shows that the Flexible Joint maximum rotational angle reduces from 22deg to 17deg after the wave particle motions are included. This changes the Flexible Joint from a challenging solution to one that is within the proven capability of the product for riser hangoff. Another SCR analysis demonstrates that the size of Titanium Taper Stress Joint (TTSJ) can be reduced if wave particle motions are included in the analysis. This phenomenon is generally explained by the “damping” effect. However, questions were never clearly answered such as how the damping works and why the damping is favorable for SCR design. This paper uses an example to demonstrate the relationships between wave particle motions and riser hangoff motions. To illustrate the behavior, analytical formulations for the vessel motions, hangoff motions, and wave particle motions are derived for a regular wave sea state. Relative motions between hangoff and wave particles are established. It is then demonstrated that the hydrodynamic force on the riser is reduced and so is the bending moment on the top section of riser.

Unlike the hangoff response which is determined mostly by motions and forces in its vicinity local to the top of the riser, TDZ response is influenced mostly by the motion at hangoff. During the motion transmission from hangoff to TDZ, the mid-section of the SCR will reduce the motions through hydrodynamic forces. It is generally understood that the SCR TDZ fatigue damage is reduced when background current is applied along riser in the motion fatigue analysis, even if the current is in the perpendicular direction to the riser motion. This paper examines the Morrison’s equation analytically and demonstrates that the current in the perpendicular direction provides damping forces in the riser motion direction.

For the motion fatigue at the riser TDZ, it is generally expected that an SCR with shorter strake length will result in higher fatigue damage compared with that of the configuration wherein the strake coverage is extended to near seabed. The expectation arises because a straked riser section has more drag force to damp the riser motion. However, as can be seen in the example of this paper, having an extended length of strake does not necessarily help reduce the riser motion. On the contrary, it reinforces the riser motion and result in larger vessel motion fatigue damage. Again, Morison’s equation is used to examine and explain the causes.

The understanding of these riser hydrodynamic behaviors helps improve the riser design. The applications of wave particles and background currents present more realistic riser responses and reduce conservatism and thus the cost. A riser with shorter strake length but larger fatigue life will result in reduced cost too, provided that adequate VIV fatigue life is preserved.

As Morison’s equation is used throughout this paper, it is presented here for ease of reference [3]. The two terms inside the parentheses are the inertia forces, including the Froude-Krylov force resulting from acceleration of the fluid itself and the added mass force due to relative acceleration between fluid and riser. The last term is the drag force.

\[
F = (Ap a_f + C_a Ap a_r) + \frac{1}{2} C_d D \rho |V_r| V_r
\]

where

\(F\): the hydrodynamic force on the riser per unit length;
\(\rho\): the fluid (sea water) density;
\(a_f\) and \(a_r\): the fluid acceleration relative to the earth and the fluid acceleration relative to the riser;
\(C_a\) and \(C_d\): the added mass coefficient and drag coefficient;
\(D\) and \(A\): the riser hydrodynamic diameter and cross-sectional area;
\(V_r\): the fluid velocity relative to riser.

**Effect of Wave Particle Motions on Riser Hangoff Section**

The effect of wave particle motions on the riser hangoff section is examined in this section using an 8inch SCR tied back to a semisubmersible by a TTSJ. For ease of demonstration and without loss of generality, the riser is attached to the centerline of the semisubmersible and the wave is applied along the vessel surge axis, as shown in Figure 1. Thus it reduces to a two dimensional problem. Two identical risers are attached to the vessel, one inside the pontoon another outside of the pontoon.

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The environmental, vessel, and riser properties are summarized in Table 1.

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<thead>
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<th>Table 1 Parameters for Examining Wave Particle Effect on Riser Hangoff</th>
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<td>Wave Amplitude (m)</td>
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<td>Wave Period (s)</td>
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<td>Wave Length (m)</td>
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<td>Vessel Far Offset (m)</td>
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<tr>
<td>Vessel Draft (m)</td>
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<td>Vessel Sway RAO Amplitude (m/m)</td>
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<td>Vessel Sway RAO Phase Lag (deg)</td>
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<td>Vessel Heave RAO Amplitude (m/m)</td>
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<tr>
<td>Vessel Heave RAO Phase Lag (deg)</td>
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<td>Vessel Pitch RAO Amplitude (deg/m)</td>
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<td>Vessel Pitch RAO Phase Lag (deg)</td>
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<td>Riser Hangoff Angle (deg)</td>
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<td>Riser Hangoff below MWL (m)</td>
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<td>Riser Hangoff X (inside Pontoon) (m)</td>
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<tr>
<td>Riser Hangoff X (outside Pontoon) (m)</td>
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<tr>
<td>Riser OD (in)</td>
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<td>Riser Wall Thickness (in)</td>
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<td>Riser Insulation Thickness (in)</td>
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<td>Riser Insulation Density (kg/m3)</td>
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<tr>
<td>Strake Thickness (in)</td>
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<td>Strake Density (kg/m3)</td>
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<td>TTSJ OD at Thick End (in)</td>
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<td>TTSJ Taper Section Length (ft)</td>
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<td>TTSJ Straight Section Length (ft)</td>
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The dynamic simulations are performed using OrcaFlex [4]. For the simulation with wave effect, a regular wave with vessel RAO is applied. For the simulation without wave effect, the 6-DOF vessel motions from the previous simulation are extracted and applied as superimposed motion. Thus, the two sets of simulations have identical vessel motions. The API-RP-2RD stress along the top of riser is shown in Figure 2. The stress time history at the critical location (bottom of TTSJ taper) is shown in Figure 3 for the duration of one wave period.
It is observed that the TTSJ stress is lower when wave particle motions are considered in the simulation. For the riser attached outside of the pontoon, the stress reduces from 725MPa to 596Mpa, or 18%. For the riser attached inside of the pontoon, the stress reduces from 694MPa to 531Mpa, or 24%. For both risers, the largest stresses occur at approximately 6.5s when wave particle motion is not considered, whereas they occur at approximately 7s when the wave particle motion is considered. Both of the times are slightly ahead of the maximum pitch (negative) occurrence time of 8s. A snapshot of the simulation is shown in Figure 4 at t=6.5s, when the vessel and riser are moving toward the right and downwards, and rotating counterclockwise.

The TTSJ stress is dominated by the bending moment which is a combined effect from vessel pitch, hydrodynamic forces, and vessel offset. From Figure 4, it can be observed that the bending moments resulting from the three sources reinforce each other. As the two sets of simulations have same vessel motion and offset, the difference in stress has to be attributed to the hydrodynamic forces. Note that for large sea states where the relative displacements between riser and water particles are larger than 0.5-1m, the drag force is larger than the inertia force [5]. More generally, the drag force dominates when the Keulegan-Carpenter (KC) number is more than 20-30, while the inertia forces dominate when the KC number is much less than 20-30 [3]. The remainder of this section discusses the drag force difference between the two simulations.
Consider a regular wave that originates at the center of the coordinate system in Figure 1 at $t = 0$ and propagates toward positive x direction, its profile can be described as

$$a \sin(\omega t - kx)$$

where
- $a$: the wave amplitude;
- $\omega$: the angular frequency given by $2\pi/T$ and $T$ the period;
- $k$: the wave number given by the dispersion relation $\omega^2 = gk$ for deep water wave.

The horizontal wave particle velocity $u$ and vertical wave particle velocity $w$ are given by [6]

$$u = \omega ae^{kx} \sin(\omega t - kx)$$
$$w = \omega ae^{kx} \cos(\omega t - kx)$$

For a semisubmersible whose center of motion is located at $x = 0, z = 0$ at $t = 0$, its displacements are determined by wave $a \sin(\omega t)$ and the RAOs,

$$X_v = a R_X \sin(\omega t - \varphi_X)$$
$$Z_v = a R_Z \sin(\omega t - \varphi_Z)$$
$$P_v = a R_P \sin(\omega t - \varphi_P)$$

where
- $X_v, Z_v, P_v$: the surge, heave, and pitch of the vessel;
- $R_X, R_Z, R_P$: RAO amplitudes for surge, heave, and pitch;
- $\varphi_X, \varphi_Z, \varphi_P$: RAO phase lags for surge, heave, and pitch.

Assuming rigid body motion of the vessel, the riser hangoff point located at $L_X$ in surge axis and $L_Z$ in heave axis can be determined by

$$X_H = X_v + L_Z \sin(P_v) + L_X \cos(P_v)$$
$$Z_H = Z_v - L_X \sin(P_v) + L_Z \cos(P_v)$$

where
- $X_H, Z_H$: the global surge and heave coordinates of the riser hangoff point;
- $L_X, L_Z$: hangoff point coordinates on the surge and heave axis relative to vessel. In this numerical example, they are -25m and -30m for the hangoff inside the pontoon, and are -45m and -30m for the hangoff outside of the pontoon.

The velocities of the hangoff point can be obtained by taking time derivative of the above equations and are not elaborated here.

The instantaneous wave particle motion at the riser hangoff point is obtained by substituting the riser hangoff coordinates $X_H$ and $Z_H$ in to the wave particle motion $u$ and $w$. 

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**Figure 4 Snapshot of Wave, Vessel and Riser Motions at 6.5s**

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For the riser attached inside of pontoon, the velocity time histories of the hangoff point and the wave particle at the
hangoff point are shown in Figure 5 for the first wave period. At time $t=6.5s$, the hangoff point velocity is 3.6m/s while the
wave particle velocity is 4.0m/s, but in different directions. If the wave particle motion is not considered, the normal relative
velocity between riser and sea water can be calculated by projecting the hangoff velocity (blue vector) onto the riser normal
direction (black vector). Note that this line is not 12deg with horizontal but varies with vessel rotation. At time $t=6.5s$, the
normal relative velocity is 3.3m/s. However, if the wave particle motion is being considered, its velocity (red vector)
projected onto the riser normal direction is 2.0m/s. Thus, the relative velocity between riser and sea water is 1.3m/s. This is a
60% reduction normal relative velocity compared with 3.3m/s. Given that the drag force is proportional to the velocity square
as given in the Morison’s equation, the reduction in drag force is 84% at the hangoff point. This velocity difference, 3.3m/s
vs. 1.3m/s as demonstrated in Figure 5, is the root cause of the stress reduction when wave particle motion is applied.

The above derivations are for the hangoff point only. For the top section below hangoff point, similar derivations can be
performed. OrcaFlex enables the output of normal relative motion and can be readily extracted. This is shown in Figure 6. It
is observed that the velocity differences are significant near the hangoff, and diminish after approximately 100m. Based on
the data of Figure 6, the total drag force over the top 100m and their bending moment to the hangoff point can be calculated.
The reductions due to wave particle motion are summarized in Table 2.

The bending moment reductions given in Table 2 are for the hangoff points of the two risers. At arc length 4.8m, they are
slightly lower. These bending moment reductions are higher than the stress reductions because they are only part of the
moments on the riser, and the dominating bending moment contribution comes from the vessel pitch.

Another observation is that the bending moment reduction is lower for the riser attached outside of pontoon. This is
because the wave particle motion at that hangoff is almost vertical at $t=6.5s$. For the riser attached outside of pontoon, the
velocity time histories of the hangoff point and the wave particle at the hangoff point are shown in Figure 7 for the first wave
period. The wave particle velocity at $t=6.5s$ is 0.3m/s toward right and 3.9m/s downwards. So its projected velocity onto the
riser normal direction is smaller. This can also be observed in Figure 4 wherein the wave particle moving directions are
indicated by blue arrows. However, the wave particle motions simulated by the software are more realistic for the riser
attached outside of the pontoon. For the riser inside of pontoon, the wave particle motions are over-predicted since the wake
effect from the pontoon is not captured.
Table 2 Drag Force and Bending Moment Reduction due to Wave Particle Motion

<table>
<thead>
<tr>
<th></th>
<th>Outside Pontoon</th>
<th>Inside Pontoon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag Force Reduction</td>
<td>44%</td>
<td>60%</td>
</tr>
<tr>
<td>Bending Moment Reduction (on Hangoff Point)</td>
<td>33%</td>
<td>45%</td>
</tr>
</tbody>
</table>

Note: Bending moment in this table is caused by drag force only.

For the vessel and riser layout in Figure 1, a random wave applied in the X direction will generate largest fatigue damages at the TDP with hot spots on the top (12 o’clock) and bottom (6 o’clock) of the riser circumference. This is because the vessel is primarily moving in plane of the riser which causes in plane bending moment variations. If a background current is applied in plane of the riser, in the positive or negative X direction, the fatigue damages is observed to decrease. This is because the
current will increase the negative work done by the hydrodynamic forces to reduce the riser motion ranges. Those are relatively easy to understand. However, what appears to be mysterious is the fact that the riser motion ranges in the plane will also be decreased and fatigue damage be reduced if the background current is applied perpendicular to the riser plane. It is not straightforward to illustrate that the riser motion ranges are reduced with perpendicular background current, as the current changes the riser motion in both in plane and perpendicular directions and riser simulation is necessary. In the remainder of this section, we demonstrate that for a given riser motion, the hydrodynamic force that resists the riser in plane motion will always increase when perpendicular background current exists. This is equivalent to the conclusion that the riser in plane motion range will decrease when the riser responds to the resisting forces.

The first step is to inspect the Morison’s equation and conclude that the inertia forces remain unchanged whether the perpendicular background current exists or not. This is because the current is given as constant speed. Thus, only drag force is under consideration.

Secondly, the effect of the drag force on a given riser motion is examined. For a steady state sinusoidal riser motion $X = \sin(t)$, the riser velocity $V_R = \cos(t)$, and the relative velocity $V_c = -\cos(t)$ assuming no current and wave. The drag force on the riser at any time is calculated as

$$\frac{1}{2} C_d \rho [\cos(t)] - [\cos(t)].$$

The work done by the drag force onto the riser during one period $T$ is

$$\frac{1}{2} C_d \rho \int_0^T \left[-\cos(t)] - [\cos(t)] \right] dsin(t).$$

This can be integrated over each quarter of the period as

$$\frac{1}{2} C_d \rho \left[ \frac{1}{3} - \frac{1}{3} - \frac{1}{3} + \frac{1}{3} \right] = -\frac{4}{3} C_d \rho .$$

The observations are: the drag force is always against riser velocity; the work done from drag force is equal in each quarter of riser motion and is always negative.

Lastly, consider a riser section that moves in the riser plane with velocity $V_R(t)$ and a perpendicular background current with velocity $V_C$, the relative velocity $V_r(t)$ can be calculated as shown in Figure 8.

$$V_c \Rightarrow V_r = \sqrt{V_c^2 + V_R^2}$$

Figure 8 Riser Velocity, Current Velocity, and Relative Velocity

If the current speed $V_c$ is zero, the drag force amplitude at any time is

$$\frac{1}{2} C_d \rho V_R^2.$$ 

Its direction is toward the left if the riser is moving toward the right in Figure 8. This drag force is always applied against the riser in plane velocity.

After applying the perpendicular current, one common error or misconception is to calculate the drag forces in the riser plane direction and the perpendicular direction separately, and then combine them to get the total drag force,

$$\frac{1}{2} C_d \rho V_R^2,$$

$$\frac{1}{2} C_d \rho V_c^2,$$

$$\frac{1}{2} C_d \rho \sqrt{V_R^4 + V_c^4}.$$ 

This will lead to the wrong conclusion that the perpendicular current speed has no effect on the in plane drag force. The fundamental reason of this approach being wrong is that the drag force amplitude does not have a linear relationship with the velocity, and the drag force must be calculated as a scalar from the resultant velocity and be applied in the opposite direction of the resultant velocity. The total drag force must be calculated per the Morison’s equation as

$$\frac{1}{2} C_d \rho V_r^2 = \frac{1}{2} C_d \rho (V_c^2 + V_R^2).$$
Its direction is per the relative velocity shown as orange arrow in Figure 8. This total drag force is then projected onto the riser plane and perpendicular directions as

\[
\frac{1}{2} C_d D \rho \left( V_c^2 + V_R^2 \right) \frac{v_R}{\sqrt{V_c^2 + V_R^2}} = \frac{1}{2} C_d D \rho V_R \sqrt{V_c^2 + V_R^2},
\]

\[
\frac{1}{2} C_d D \rho \left( V_c^2 + V_R^2 \right) \frac{v_c}{\sqrt{V_c^2 + V_R^2}} = \frac{1}{2} C_d D \rho V_c \sqrt{V_c^2 + V_R^2}.
\]

It can be observed that the drag force in the riser plane is always larger than the case where current speed is zero. Since the drag force is always applied against the riser motion and always does negative work onto the riser, the in plane drag force with perpendicular current will do more work to resist the riser motion, compared with the no current load condition.

Reducing Strake Length to Increase Motion Fatigue Life

Strakes are effective tool for suppressing riser VIV responses especially for areas with high currents. Generally, the riser motion fatigue life can also benefit from long strake coverage due to its larger drag coefficient to damp the riser motion. Thus, many risers are straked toward or beyond the Touch Down Point (TDP). However, some recent studies have found that decreasing the strake coverage at the TDP area does not necessarily decrease the motion fatigue life. On the contrary, shorter strake coverage, with the strakes stopping just short of the TDP could reduce the riser motion range and increase the TDP fatigue life.

A numerical example is studied to explain this phenomenon wherein the fatigue damage is reduced by approximately 20% after removing a short section of strake near the TDP. Similar to the previous section, it is not straightforward to analytically illustrate that the riser motion ranges are reduced when strake is removed, as it changes the riser motion and riser simulation is necessitated. In the remainder of this section, we demonstrate how the hydrodynamic forces change and their effects on the riser, when strake is removed.

The key parameters relevant to this assessment are given in Table 3, with the riser, environment, and vessel layout same as those in Figure 1. One riser is straked for the top 1807m, another straked for the top 1777m, a 30m difference. The riser motion amplitude in Table 3 is obtained at the straked section near TDP, from the OrcaFlex simulation. It is normal to the riser axis. The sea water near seabed is assumed to be still.

<table>
<thead>
<tr>
<th>Table 3 Parameters for Examining Strake Removal near TDP</th>
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<td>Water depth (m)</td>
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<td>Wave Height (m)</td>
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<td>Wave Period (s)</td>
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<tr>
<td>Riser Hangoff X (m)</td>
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<tr>
<td>Riser OD (in)</td>
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<tr>
<td>Riser Wall Thickness (in)</td>
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<tr>
<td>Riser Insulation Thickness (in)</td>
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<tr>
<td>Riser Insulation Density (kg/m3)</td>
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<tr>
<td>Strake Thickness (in)</td>
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<tr>
<td>Strake Density (kg/m3)</td>
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<tr>
<td>Internal Fluid Density (kg/m3)</td>
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<tr>
<td>Drag Coefficient, Strake</td>
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<tr>
<td>Added Mass Coefficient, Strake</td>
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<tr>
<td>Drag Coefficient, Bare</td>
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<tr>
<td>Added Mass Coefficient, Bare</td>
</tr>
<tr>
<td>Motion Amplitude at Arc Length 1792m (m)</td>
</tr>
</tbody>
</table>

Based on the riser motion amplitude and period, its peak velocity and acceleration are 0.0416m/s and 0.0327m/s². The hydrodynamic diameter is 0.451m for strake and 0.425m for bare riser. The hydrodynamic forces can be calculated based on the Morison’s equation and are given in Table 4. Note that since the sea water is assumed to the still near the seabed, the water particle acceleration \( a_f \) is zero and thus the Froude-Krylov force is zero.

<table>
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<th>Table 4 Maximum Drag Force and Added Mass Force (N/m)</th>
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<tr>
<td>Drag Force</td>
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<td>Straked Riser</td>
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<tr>
<td>Bare Riser</td>
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</table>
It is observed that the amplitude of added mass force is an order of magnitude larger than the drag force, for both straked and bare risers. This is consistent with the conclusion in [5][3] and mentioned in the wave particle motion section. Another observation is that both of the hydrodynamic forces are lowered by approximately half after the strake is removed which is primarily due to the change in drag and added mass coefficients.

The reduction in drag force, as illustrated in last section, will decrease the resistance force and thus tends to increase the riser motion range and thus fatigue damage. However, their magnitudes are very small in this example. The effect of reduction in added mass force is unclear to this point. Similar to the last section, we examine the effect of added mass force by calculating the work it does to the riser during one period.

For a steady state sinusoidal riser motion \( X = \sin(t) \) in still water, the riser acceleration is \( a_R = -\sin(t) \), and the relative acceleration \( a_r = \sin(t) \). The added mass force on the riser at any time is calculated as

\[
C_a \rho \frac{A \omega^2}{2} \frac{\sin(t)}{t}
\]

The work done by the added mass force onto the riser during one period \( T \) is

\[
C_a \rho \int_0^T \sin(t) ds \sin(t)
\]

This can be integrated over each quarter of the period as

\[
C_a \rho \left[ \int_0^{T/4} \sin(t) ds \sin(t) + \int_{T/4}^{T/2} \sin(t) ds \sin(t) + \int_{T/2}^{3T/4} \sin(t) ds \sin(t) + \int_{3T/4}^T \sin(t) ds \sin(t) \right]
\]

After some calculus, the work done by the added mass force is,

\[
C_a \rho \left[ \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \right] = 0.
\]

Thus, the net work done by the added mass force is null to the riser with given sinusoidal motion. So, the above derivations do not help determine the effect of added mass force on a riser that responds to it.

We now look into the general explanation of the added mass force. As demonstrated in [7], for an accelerating body (riser), all of the fluid around the riser is accelerating to some degree such that the total kinetic energy of the fluid is increasing due to the acceleration of riser. The added mass force has the same form and sign as that required to accelerate the mass \( m \) of the body itself. Consequently, it is often convenient to visualize the mass of the associated fluid as an “added mass” \( M \) of fluid which is being accelerated “with” the body. It was derived in [7] that in a potential flow the added mass of the cylinder is equal to the mass of the fluid displaced by the body and the added mass of a sphere is one half of the displaced mass. Thus, for a riser under still water, its mass can be visualized as \( m + M \). In fact, some of the software such as Flexcom [8] does not integrate the added mass force into the right hand side force vector of the equation of motion; instead the added mass is taken to the left hand side of the equation of motion and integrated into the mass matrix.

The total mass including the riser body (steel pipe, insulation, internal fluid, and strake where appropriate) and the added mass is given in Table 5. The mass of riser body is similar whether it is straked or bare. The added mass for the bare riser is approximately 50% of its riser mass, whereas the added mass for the straked riser is close to its riser mass due to its large added mass coefficient. Overall, the total mass of the straked riser is 43% larger compared with the bare riser. It is worth emphasizing that the added mass does not contribute to the weight of the riser. In other words, the gravity to the riser only considers the mass of the riser and not the added mass.

| Riser Mass Added Mass Total Mass |
|---------------------------|-----------------|----------|
| Straked Riser 337 327 664 | Bare Riser 319 145 464 |

This 30m riser section, from a structural dynamics point of view, loses 200kg/m of mass after removing its strake. In dynamic motions, its inertia, the resistance of changing its state of motion, decreases while the inertia of other sections remains the same. This relative change in inertia affects the motion characteristics of the displacement controlled riser system, results in reduced motion in this 30m section but increased motion in its neighboring section. As shown in Figure 9, the displacement amplitudes in the immediate vicinity of this 30m section (from arc length 1777m to 1807m) become smaller, whereas the amplitudes further away from this section become larger. Since this reduction in displacement amplitude is right before the TDP (at arc length of approximately 1815m), the TDP curvature and bending moment variations, hence the fatigue damage is reduced.
The effect of the reduced total mass on riser motion is explained qualitatively in the previous paragraph based on structural dynamics interpretation. It can also be demonstrated quantitatively from the riser modal responses. Several mode shapes of the two risers are shown in Figure 10 and Figure 11 for in plane modes and out of plane modes, respectively. Similar to the observations from Figure 9, the modal displacements before the TDP are all smaller for the riser with the 30m strakes removed, indicating reduced curvatures and fatigue damage.
Riser Design Improvement and Cost Reduction

Wave particle motions have large impact on the responses of riser hangoff section in large sea states. The effect is even larger if the hangoff point is closer to the sea surface. For an SCR tied back to a Tension Leg Platform (TLP) where the porch is only 15m below Mean Water Level (MWL), a stress reduction of more than 30% was observed. Thus, it is important that the wave particle motions be included in the riser simulation. In most analyses of SCR detailed design, the vessel motions are applied using time traces to capture its offsets and first and second order motions. It is recommended that the corresponding waves be applied, through either harmonic components or sea surface elevations, to reduce the conservativeness especially when the riser is attached outside of pontoon. The more realistic analysis results will decrease the TTSJ stress or the Flexible Joint rotation angle. This may change the feasibility of the SCR attachment methods, or it will reduce the sizes of the components. Therefore, it is expected that the cost of SCR can be lowered.

Background currents exist in most sea states. Thus, it is realistic to include currents to the riser motion fatigue analyses preferably the associated current with each sea state. As presented in this paper, even when the current direction is perpendicular to the vessel/riser motion direction, the riser motion range and thus fatigue damage will reduce due to the increased drag force. The more realistic simulation and larger fatigue life may change the feasibility of the SCR, or may lower its construction requirements, and thus reduce the cost.

In fields without much of bottom currents, a riser with shorter strake length can be used to improve fatigue life. In addition, its TDP strength response with shorter strake will also improve in extreme environments due to reduced drag forces [5]. The shorter strake length will of course reduce the cost for material, construction and installation. The enhanced strength and motion fatigue performances may also change the feasibility or construction requirements, and reduce the cost further.

Conclusions

This paper presented the solutions to three mysterious SCR hydrodynamic behaviors and provided recommendations accordingly for riser design improvement and cost reduction. At the top of the SCR, the TTSJ maximum stress or the Flexible Joint maximum rotational angle can be significantly lower when the wave particle motion is considered. By examining the motions of vessel, porch, and wave particles, it was demonstrated that the wave particle is moving in phase with the motion of the riser top section when the maximum stress or rotational angle occurs. This reduces the relative velocity between riser and sea water, and subsequently the drag force and bending moment.

The background current in the middle of SCR can reduce the riser motion range and thus the TDZ fatigue damage, even if the current direction is perpendicular to the riser motion direction. This is because of increased relative velocity and drag force which always does negative work (or damping) to the riser motion.

The inertia force is several times larger than the drag force when the riser motion is small such as at the TDZ area in moderate or small sea states. The inertia of the section near the TDP changes the SCR motion characteristics, results in smaller motion and larger fatigue life when strakes are removed. The understandings of these riser hydrodynamic behaviors can all the utilized to improve the riser design. These principles can also be applied to similar structures such as umbilicals and flexible risers.

References